Open problems and career in mathematics

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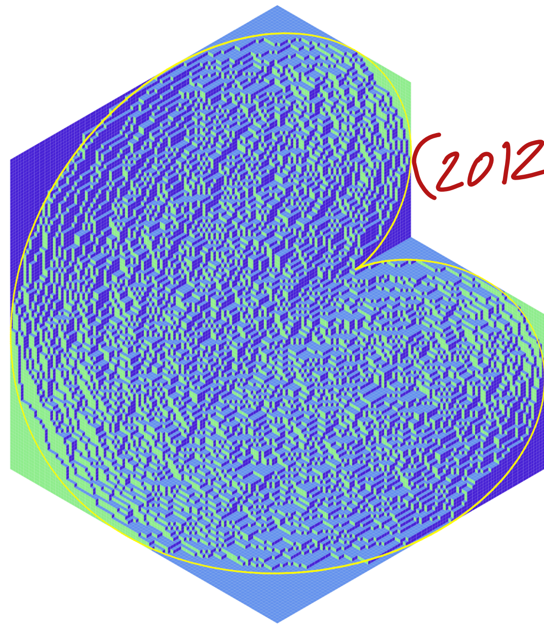
$$\overline{\nabla}_\infty = \left\{ x = (x_1, x_2, \dots) \in [0,1]^\infty : x_1 \geqslant x_2 \geqslant \dots \geqslant 0,\ \sum_{i=1}^\infty x_i \leqslant 1 \right\}.$$

In the topology of coordinatewise convergence $\overline{\nabla}_\infty$ is a compact, metrizable and separable space. Denote by $C(\overline{\nabla}_\infty)$ the algebra of real continuous functions on $\overline{\nabla}_\infty$ with pointwise operations and the supremum norm.

In $C(\overline{\nabla}_\infty)$ there is a distinguished dense subspace $\mathcal{F} := \mathbb{R}[q_1, q_2, \dots]$ generated (as a commutative unital algebra) by algebraically independent continuous functions $q_k(x) := \sum_{i=1}^\infty x_i^{k+1}$, $k = 1, 2, \dots$, $x \in \overline{\nabla}_\infty$.

For each $0 \leqslant \alpha < 1$ and $\theta > -\alpha$ we define an operator $A \colon \mathcal{F} \to \mathcal{F}$ which can be written as a formal differential operator of second order with respect to the generators of the algebra $\mathcal{F}$:[1]

$$(1)\qquad \begin{aligned} A \;=\; & \sum_{i,j=1}^\infty (i+1)(j+1)(q_{i+j} - q_i q_j) \frac{\partial^2}{\partial q_i \partial q_j} \\ & + \sum_{i=1}^\infty \left[-(i+1)(i+\theta) q_i + (i+1)(i-\alpha) q_{i-1} \right] \frac{\partial}{\partial q_i}, \end{aligned}$$



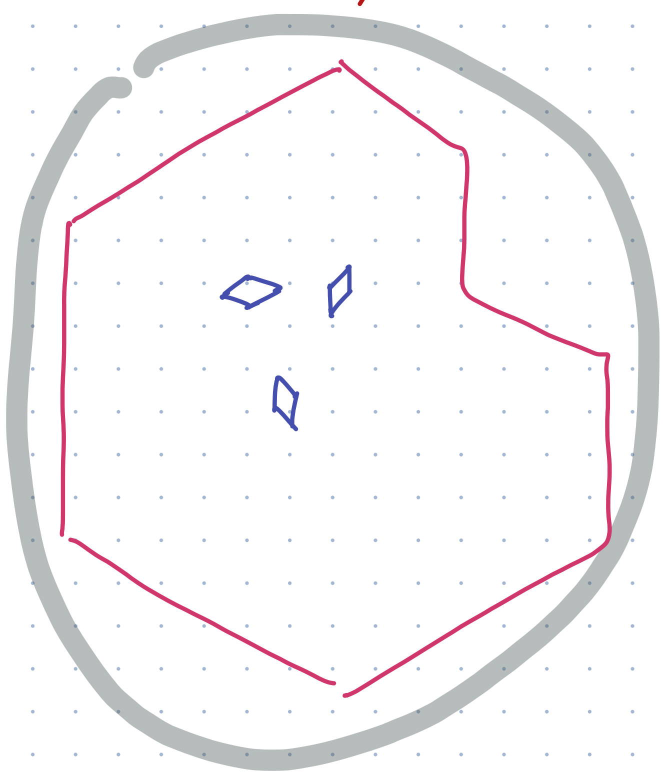
(2012)

Proposition 3.1. *Consider the transition operator of the nth up/down Markov chain $T_n \colon \mathrm{Fun}(\mathbb{K}_n) \to \mathrm{Fun}(\mathbb{K}_n)$ which corresponds to the two-parameter Ewens-Pitman's partition structure. Its action on the functions $\mathbf{m}^*_\mu$, $\mu \in \mathbb{K}$ looks as follows:*

$$\begin{aligned} (T_n - \mathbf{1})(\mathbf{m}^*_\mu)_n = & -\frac{k(k-1+\theta)}{(n+1)(n+\theta)} (\mathbf{m}^*_\mu)_n \\ & + \frac{n+1-k}{(n+1)(n+\theta)} \sum_{\substack{i=1 \\ \mu_i \geqslant 2}}^{\ell(\mu)} \mu_i (\mu_i - 1 - \alpha) \left(\mathbf{m}^*_{\mu - \square(\mu_i)} \right)_n \\ & + \frac{n+1-k}{(n+1)(n+\theta)} [\mu : 1] \left(\theta + \alpha(\ell(\mu) - 1) \right) \left(\mathbf{m}^*_{\mu - \square(1)} \right)_n, \end{aligned}$$

where $\mathbf{1}$ *denotes the identity operator and* $k = |\mu|$.

(2006-7)

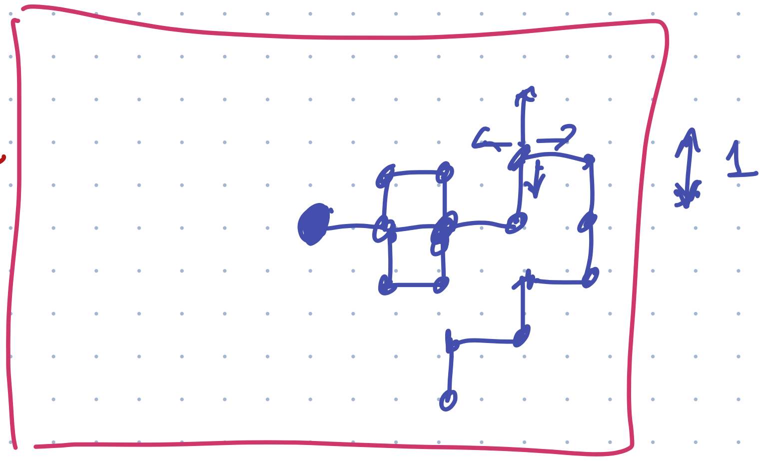


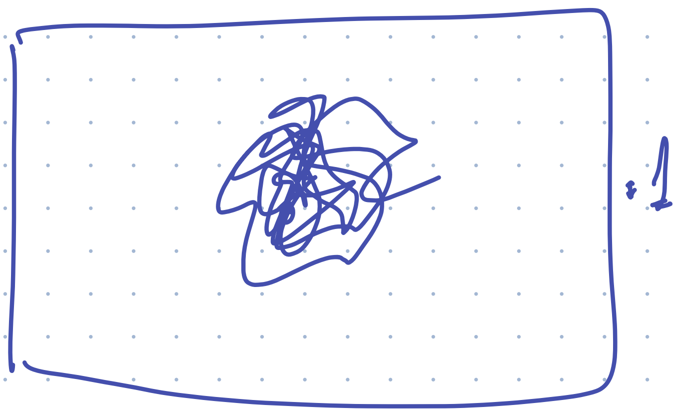
(Integrable Probability) $\cap$ Probability

Самый красивый матем. объект / факт / теорема

— фракталы

самоподобные фигуры





— ряды Тейлора / производ. ф.

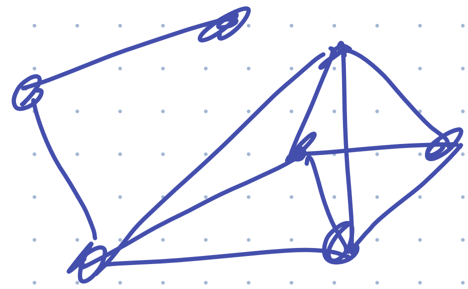
$$\forall f(x) \simeq \boxed{\text{полином}}$$

— $$\boxed{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} \longrightarrow \infty, \quad n \to \infty}$$

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n}\sim\boxed{\ln n}$$

$\boxed{100}$ e^{100}

— квази-случайные графы $\binom{2}{\emptyset}$



Мат. образование vs Research

Мат. образование	Research
— бакалавр / магистр (-40к/год)	~~— доказ. теоремы~~
— Grad school (PhD) (+30 к/год)	~~— писать статьи~~ (рецензир.)
— Post Doc (временная работа) 60–70	— учить
— Professor 30+ …	— делать доклады, ездить на конф. орг. конф.

— гранты (писать заявки)
— админ. работа

Grad School

5–6 лет
— учиться
— писать статьи
— преподавать

⇓

PhD

US / Canada

US News rating

Топ 10

UCLA
MIT / Harvard
Stanford
Princeton
Yale
Berkeley...

15 — 50 OK level

(University of Virginia)

(о) есть уже 1 статья / преприн

1) рекомендации (2-3) reading course

2) курсы по математике

(Европа ✓

US/Can : аккр. курс)

оценки

3) REU / летний research

4) Английский